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Mathematical modeling of cavitation-free electrochemical machining process

Modelowanie matematyczne procesu elektrochemicznej obróbki materiału z uwzględnieniem zjawiska kawitacji

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Electrochemical machining of machine elements is modeling in two-dimensional formulation, Influence of shaped section of the cathode on cavitation phenomenon was taken into account. The aim of investigations was to determine machining conditions provided cavitation-free electrolyte flow in the space of large pressure gradients.

KEYWORDS: electrochemistry, treatment of material, potential, electrolyte cavitation, cathode.

W dwuwymiarowym polu modelowano matematycznie proces elektrochemicznej obróbki części maszyn uwzględniając wpływ profilowanej części roboczej katody na zjawisko kawitacji. Celem badań było określenie warunków obróbki zapewniających bezkawitacyjny przepływ elektrolitu w przestrzeni dużych gradientów ciśnień.

SŁOWA KLUCZOWE: elektrochemia, obróbka materiału, potencjał, kawitacja elektrolitu, katoda.

Introduction

In this paper a mathematical model of the electrochemical machining the flywheel car is built. When we treat a part in stationary mode, at the corner points (point B in Figure 1) the flow velocity increases sharply and as a consequence there are arising cavities, fulfilled by air bubbles. In these areas electrical conductivity of electrolyte is broken and they play the role of insulator. This leads to the formation of irregularities on the surface to be treated.

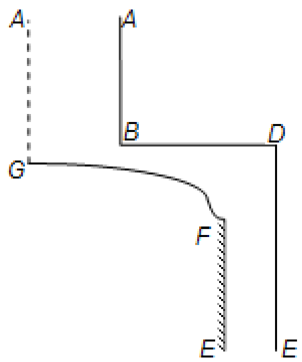

Fig. 1. Interelectrode gap before modification

To eliminate this phenomenon one proposed to replace the corner points on the cathode by a shaped sections, which ensure smooth flow (BC in Figure 2). It was used methods of the theory of jet streams [1] what enable to build analytical solution of the problem.

Problem formulation

Figure 2 show the plane sections (right half) of the electrode gap (IEG). We use complex coordinate $z = x + iy$. Here AG is the line of symmetry, $ABCDE$ is the boundary of the cathode, GF is the anode boundary, EF is the electrically insulated section of the IEG, A, E are points at infinity. The origin is chosen at the point G. The feed rate of

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the cathode $\vec{V}_K$ is orthogonal to axis x, directed along the edge CD. The electrolyte flow in the IEG direct from the point A to point E. The IEG width in neighborhood of points A and E are H_A and H_E electrolyte flow speed are V_A and V_E. The electrolyte flow speed on the profiled edge is constant and equal V_0.

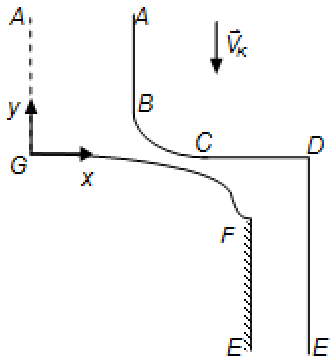

Fig. 2. Elements of new IEG area

Condition of stationarity of electrochemical shaping is true on the border of the anode. Our goal is to determine the shape of anode surface and cathode profiled section.

Mathematical modeling

We assume that the rate of metal removal V_m from the anode surface per unit mass, determined by Faraday's law $V_m = j_A \eta \varepsilon$, where $\eta = \eta(j_A)$ is the current efficiency, j_A is the current density, ε is the metal electrochemical equivalent, cathode surface moves with constant velocity and linear velocity of points on the surface of the anode is

$$V_A = V_K \cdot \cos\theta\,, \qquad (1)$$

where θ is the angle between the feed rate direction and the unit outward normal to the anode surface. In this case, the general scheme of the process does not change with time, and the process can be regarded as stationary. Steady current density distribution on the stationary anode j_A is defined as:

$$\eta j_A = \frac{\rho V_K}{\varepsilon}\cos\theta\,,$$

where ρ is density of the anode material. We assume that at the boundary of the anode, the next relation is true [2, 3]

$$j_A = \frac{\rho V_K}{\varepsilon}(a_0 + b_0\cos\theta) \qquad (2)$$

Consider a two-dimensional model of the process (Figure 2). We introduce a Cartesian coordinate system x_1, y_1, associated with anode.

We assume neglecting the near-electrode phenomena in the IEG there is electric field potential ψ_1, satisfying the Laplace equation

$$\Delta\psi_1 = 0 \qquad (3)$$

and on the electrode borders conditions of constant potential $\psi_{1A} = u_A$, $\psi_{1K} = u_K$ are true. By (3), there exists a function φ_1, harmonically conjugate to ψ_1, and we can enter the complex potential of the electrostatic field $W(x_1, y_1) = \varphi_1(x_1, y_1) + i\psi_1(x_1, y_1)$, which is an analytic function in area $z_1 = x_1 + iy_1$.

We introduce the characteristic values of the current density $j_0 = \rho V_K/\varepsilon$, length $H = \kappa(u_A - u_K)/j_0$ (κ - electrical conductivity of the medium) and move on to the dimensionless variables

$$x = \frac{x_1}{H}\,,\quad y = \frac{y_1}{H}\,,\quad z = x + iy\,,\quad W = \varphi + i\psi = \frac{W_1 - iu_K}{u_A - u_K}\,.$$

Then, in view of (2) function ψ satisfies in IEG to Laplace equation and the boundary conditions on electrode surfaces.

$$\psi_A = 1\,,\quad \psi_K = 0\,,\quad \frac{\partial\psi_A}{\partial n} = \frac{j_A}{j_0} = a_0 + b_0\cos\theta\,,$$

where, a_0, b_0 are constants, taking into account the dependence current output of the current density.

The variation area of the electrostatic complex potential W is a rectangle $D_W = \{\varphi + i\psi,\ 0 \le \psi \le 1,\ 0 \le \varphi \le \varphi_0\}$ (Figure. 3).

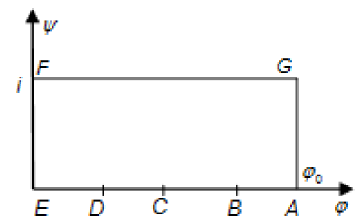

Fig. 3. Domain D_W

Let in the plane of the auxiliary complex variable $u = \xi + i\tau$ region $D_u = \{u = \xi + i\eta, 0 \le \xi \le \pi/2, 0 \le \eta \le \pi\tau/4\}$ corresponds to the area flow D_z (Figure 4), and the function $z(u)$ conformally maps the domain D_u to the domain D_z with the points correspondence, indicated in Figure 2, 4.

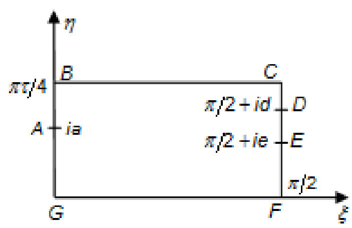

Fig. 4. Domain D_u

We define two functions: complex potential of the electrolyte flow [4] $W_g(u) = \varphi_g(u) + i\psi_g(u)$ and the function of Zhukovsky [1]

$$\chi(u) = \ln\left(\frac{V_0 dz}{dW_g}\right) = r(u) + i\theta(u) \tag{4}$$

where $r(u) = \ln(V_0/V)$, V is velocity of the electrolyte flow, θ is the angle of the velocity vector of the axis x.

Complex potential of electrolyte flow $W_g(u)$ satisfies the boundary conditions

$$\begin{aligned} \operatorname{Im} W_g(u) = \psi_g(u) = 0, \quad & u = i\eta,\ 0 \le \eta < a;\ u = \xi; \\ & u = \frac{\pi}{2} + i\eta,\ 0 \le \eta < e; \\ \operatorname{Im} W_g(u) = \psi_g(u) = q, \quad & u = i\eta,\ a < \eta \le \frac{\pi|\tau|}{4};\ u = \xi + \frac{\pi\tau}{4}; \\ & u = \frac{\pi}{2} + i\eta,\ e < \eta \le \frac{\pi|\tau|}{4} \end{aligned}$$

The variation area of the function $W_g(u)$ is the strip $D_{W_g} = \{\varphi_g + i\psi_g,\ 0 \le \psi_g \le q\}$, $q = V_E H_E$ is the electrolyte flow output.

The boundary conditions for the complex potential $W_g(u)$ let us to construct function $W_g(u)/du$ by the singular points method [1]:

$$\frac{dW_g(u)}{du} = N \frac{\vartheta_1(2u)}{\vartheta_1(u-ia)\vartheta_1(u+ia)\vartheta_4(u-ia)\vartheta_4(u+ia)} \times \frac{\vartheta_4(2u)}{\vartheta_2(u-ie)\vartheta_2(u+ie)\vartheta_3(u-ie)\vartheta_3(u+ie)}$$

where $\vartheta_i(u)$ ($i = \overline{1,4}$) is theta functions for periods π and $\pi\tau$ [5]. Constant N is determined from the condition that the flow output is q. Determining the residue of the function $W_g(u)$ at the point A, we obtain

$$N = \frac{q}{\pi}\vartheta_1'(0)\vartheta_4(0)\vartheta_2(ia-ie)\vartheta_2(ia+ie)\vartheta_3(ia-ie)\vartheta_3(ia+ie).$$

Zhukovsky function $\chi(u) = r(u) + i\theta(u)$ satisfy the following boundary conditions

$$\begin{aligned} \operatorname{Im}\chi(u) = \theta(u) = -\frac{\pi}{2}, \quad & u = i\eta;\ u = \frac{\pi}{2} + i\eta,\ 0 < \eta < d; \\ \operatorname{Im}\chi(u) = \theta(u) = 0, \quad & u = \frac{\pi}{2} + i\eta,\ d < \eta \le \frac{\pi|\tau|}{4}; \\ \operatorname{Re}\chi(u) = r(u) = 0, \quad & u = \xi + \frac{\pi\tau}{4}. \end{aligned}$$

On anode border GF there is condition

$$r(\xi) = \ln\left(\frac{V_0}{a_0 + b_0\cos\theta(\xi)}\frac{d\varphi(\xi)}{d\varphi_g(\xi)}\right),$$

which allows to take into account the regime of the electrolyte flow and variability of the current output.

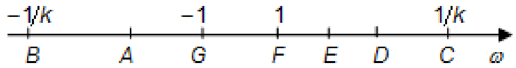

Fig. 5. Domain D_ω

To determine the function $d\varphi/d\varphi_g$ area D_u is displayed on the upper half-plane D_ω with the points correspondents, indicated in Figure 5, with help of transform

$$\omega(u) = sn\left(2K\left(\frac{2u}{\pi} - \frac{1}{2}\right), k\right) = -\frac{1}{\sqrt{k}}\frac{\vartheta_2(2u)}{\vartheta_3(2u)}, \quad k - \frac{\vartheta_2^2(0)}{\vartheta_3^2(0)} \tag{5}$$

And, using the Schwarz-Christoffel formula [6], we find the derivatives of functions mapping the area D_ω to variation areas of functions W, W_g:

$$\frac{dW}{d\omega} = \frac{M}{\sqrt{(\omega^2-1)(\omega-\alpha)(\omega-\sigma)}},$$

$$M = \left(\int_{\omega(ia)}^{-1} \frac{d\omega}{\sqrt{(\omega^2-1)(\omega-\omega(ia))(\omega-\omega(\pi/2+ie))}}\right)^{-1},$$

$$\frac{dW_g}{d\omega} = \frac{q}{\pi}\frac{\sigma-\alpha}{(\omega-\alpha)(\omega-\sigma)},$$

where $\alpha = \omega(ia)$, $\sigma = \omega(\pi/2+ie)$.

Using these formulas, we find

$$\frac{d\varphi(\xi)}{d\varphi_g(\xi)} = \frac{M\pi}{q(\sigma-\alpha)}\frac{\sqrt{(\omega(\xi)-\alpha)(\omega(\xi)-\sigma)}}{\sqrt{\omega^2(\xi)-1}}.$$

Zhukovsky function $\chi(u)$ we find as in the form $\chi(u) = \chi_0(u) + f(u)$, where the function $\chi_0(u) = r_0(u) + i\theta_0(u)$ satisfies the boundary conditions:

$$\begin{aligned} \operatorname{Im}\chi_0(u) = \theta(u) = -\frac{\pi}{2}, \quad & u = i\eta;\ u = \frac{\pi}{2} + i\eta,\ 0 < \eta < d; \\ \operatorname{Im}\chi_0(u) = \theta(u) = 0, \quad & u = \xi;\ u = \frac{\pi}{2} + i\eta,\ d < \eta \le \frac{\pi|\tau|}{4}; \\ \operatorname{Re}\chi_0(u) = r(u) = 0, \quad & u = \xi + \frac{\pi\tau}{4} \end{aligned}$$

and has in D_u the same singularities as that for $\chi(u)$. Function $f(u)$ is an analytical at D_u and continuous in $\overline{D}_u$. By the method of singular points we have [1]:

$$\chi_0(u) = \ln\left(\frac{\vartheta_4(u)\vartheta_2(u)}{\vartheta_1(u)\vartheta_3(u)}\right) - \frac{1}{2}\ln\left(\frac{\vartheta_2(u-id)\vartheta_2(u+id)}{\vartheta_3(u-id)\vartheta_3(u+id)}\right).$$

Comparing the boundary conditions for the functions $\chi(u)$ and $\chi_0(u)$, we obtain the boundary conditions for the unknown function $f(u) = \lambda(u) + i\mu(u)$:

$$\begin{aligned}&\operatorname{Im} f(u)=\mu(u)=0, \quad u=i\eta;\ u=\frac{\pi}{2}+i\eta;\\ &\operatorname{Re} f(u)=\lambda(u)=0, \quad u=\xi+\frac{\pi\tau}{4};\end{aligned} \tag{6}$$

$$\begin{aligned}&\operatorname{Re} f(u)=\lambda(u)=\\ &=\ln\left(\frac{V_0}{a_0+b_0\cos(\theta_0+\mu(u))}\frac{d\varphi(u)}{d\varphi_g(u)}\right)-r_0(u), \quad u=\xi\end{aligned} \tag{7}$$

To construct the unknown function $f(u)$ area D_u is displayed on the semicircle with help of the function [7]

$$t=e^{2i\left(u-\frac{\pi\tau}{4}\right)}$$

Taking into account the boundary conditions (6) and (7), the function $f(u)$ can be analytically continued to hole ring and to write in the form of a Laurent series:

$$f(u)=\sum_{n=-\infty}^{\infty} c_n u^n\ ,$$

where c_n are real coefficients.

On the basis of the boundary conditions (6) we find: $c_0=0$, $c_n=c_{-n}$, and obtain

$$f(u)=2i\sum_{n=1}^{\infty} c_n \sin\left(2n\left(u-\frac{\pi\tau}{4}\right)\right).$$

Condition (7), taking into account the representation of a function $f(u)$ by the series, has the form:

$$\begin{aligned}&2\sum_{n=1}^{\infty} c_n\cos(2n\xi) sh\left(\frac{\pi|\tau|n}{2}\right)=\\ &=\ln\left(\frac{V_0}{a_0+b_0\cos(\theta_0+\mu(\xi))}\frac{d\varphi(\xi)}{d\varphi_g(\xi)}\right)-r_0(\xi)\end{aligned} \tag{8}$$

Multiplying equation (8) for $\cos(2n\xi)$ and integrating by ξ within $0, \pi/2$, we obtain the infinite system of equations for the coefficients c_n:

$$\begin{aligned}&c_n=\frac{2}{\pi sh(\pi|\tau|n/2)}\times\\ &\times\int_0^{\pi/2}\left(\ln\left(\frac{V_0}{a_0+b_0\cos(\theta_0+\mu(\xi))}\frac{d\varphi(\xi)}{d\varphi_g(\xi)}\right)-r_0(\xi)\right)\cos(2n\xi)d\xi,\\ &n=\overline{1,\infty}\end{aligned}$$

Dimensionless coordinates of the anode boundary and cathode profiled section is determined from (4) by the formula

$$z(u)=\frac{1}{V_0}\int_0^u \frac{dW_g}{du}e^{\chi(u)}du\ .$$

To solve the problem it is necessary to determine the mathematical parameters τ, a, d e. It can be done by setting the speed ratios V_A/V_0, V_E/V_A, and anode dimensions.

Results of analysis

Figure 5 and Figure 6 show some of the calculation results. As It can be seen from the figures, the anode surface in the area, close to the profiled section on cathode, varies monotonically.

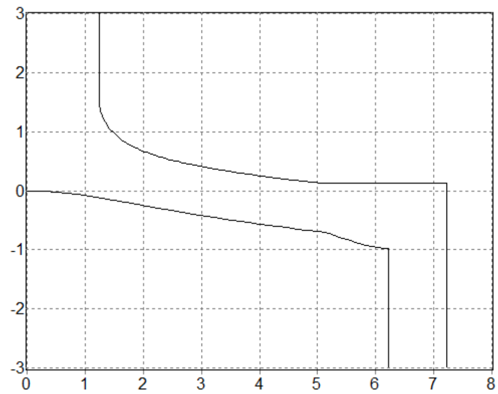

Fig.5. $a_0=0.1$, $b_0=0.9$; $L=6.2149$, $h=0.9828$; $V_0/V_A=1.5625$, $V_E/V_A=1.2045$

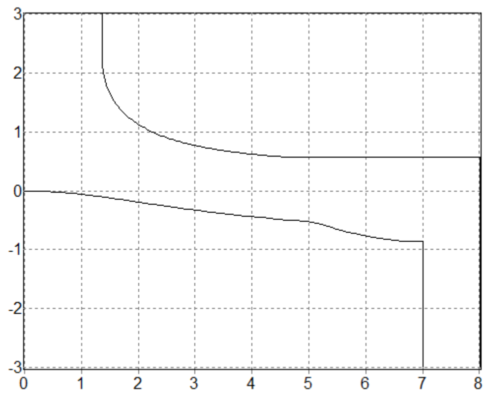

Fig.6. $a_0=0$, $b_0=1$; $L=7.009$, $h=0.8559$; $V_0/V_A=1.3449$, $V_E/V_A=1.5721$

Closure

The results of calculations: a_0, b_0 are constants, reflected the dependence of the current efficiency on the current density; $L=\operatorname{Re}(z(\pi/2))$, $h=-\operatorname{Im}(z(\pi/2))$ are "length" and "height" of the anode; V_0/V_A is ratio of the velocities, V_0 is the velocity of the electrolyte flow on the profiled section, V_A is the flow velocity at the point A; $V_E/V_A=H_A/H_E$ is the ratio of velocities at points A and E.

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