# Mathematical model of globoid worm with concave and convex tooth profile

## Matematyczny model ślimaka globoidalnego o wklęsłym i wypukłym zarysie zęba

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***This paper presents a mathematical model of globoid worm with concave and convex tooth profile. The method of creating the parametric equation of the profile was shown. The parametric equation of teeth surfaces of globoid worm with no-straight tooth profile was obtained.***

***KEYWORDS: globoid worm gear, globoid worm***

When describing the geometry of a globoid worm, it should be started from determining the profile of the worm thread [1] - it may be, for example, rectilinear, concave or convex. The mathematical model and the CAD model of a globoid worm with no-straight tooth profile are presented in [2, 3]. Making the model of a globoid worm with no-straight tooth profile is the basis for the model of the wormwheel and for carrying out selected analyzes of the gear, such as the contact pattern analysis. Fig 1 shows a graphical overview of the globoid worm gear (worm and wormwheel).

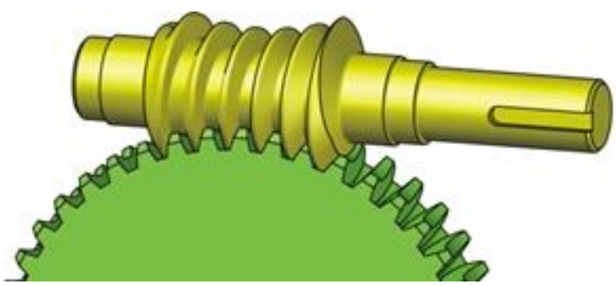

Fig 1. Illustration of a globoid worm gear fragment [4]

### Mathematical model of globoid worm with no-straight tooth profile

The concave and convex profile of the globoid worm is bounded by points *A* and *B* for one flank and *C* and *D* for the other flank (Fig 2). Coordinates of points $A(y_A, z_A)$, $B(y_B, z_B)$, $C(y_C, z_C)$ and $D(y_D, z_D)$ depend on the parameters of the gear. The way they are determined was discussed in the paper [2].

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After defining the coordinates of the contour points for one and the other flank of the tooth, it is necessary to use the parametric equation of the circle with the radius R:

$$(y - y_o)^2 + (z - z_o)^2 = R^2 \qquad (1)$$

where: *y, z* – coordinates of the point located on the circle; $y_o$, $z_o$ – coordinates of circle center, *R* – circle radius.

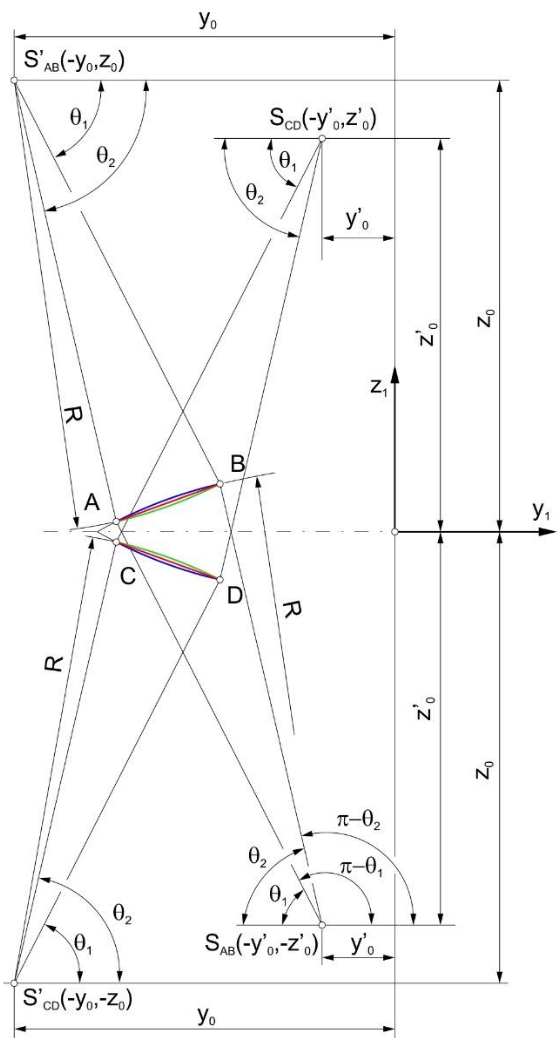


Fig 2. Auxiliary drawing for defining the arc center of a given profile and the range *θ* for concave and convex profile of the globoid worm. Designations: *A, B* or *C, D* – end points of the arc; $y_o$, $z_o$ ($y'_o$, $z'_o$) – coordinates of the center of arc *S* or *S'* of a given profile; *R* – arc radius; $\theta_1$, $\theta_2$ – angular values used to determine the range *θ* of the arc profile

To determine the coordinates of the center of the arc profile, system of two equations (1) should be created, substituting the coordinates of points *A* and *B* for one flank and *C* and *D* for the other flank subsequently for *y* and *z*. After solving the system of equations, two solutions of the circle center *S* are obtained. The choice of the solution determines then the character of the profile - concave or convex.

The next step is to determine the angular range of the profile. Angular values $\theta_1$ and $\theta_2$ should be defined (Fig 2), which can be determined, e.g. from the dependence:

$$\begin{aligned}\theta_1 &= arcsin\left((z_o - z_B)/R\right)\\ \theta_2 &= arcsin\left((z_o - z_A)/R\right)\end{aligned} \tag{2}$$

Then the range $\theta$ for the given arc is determined (Fig 2). The angular parameter of the arc $\theta$ changes from the initial value $\theta_p$ to the final value $\theta_k$ with the step d$\theta$.

In the case of the *AB* curve, for which a concave thread (green) is obtained, the range $\theta$ is:

$$\begin{aligned}\theta_p &= \theta_2\\ \theta_k &= \theta_1\end{aligned} \tag{3}$$

In the case of the *CD* curve, for which a concave thread (green) is obtained, the range $\theta$ is:

$$\begin{aligned}\theta_p &= \theta_1\\ \theta_k &= \theta_2\end{aligned} \tag{4}$$

In the case of the *AB* curve, for which a convex thread (blue) is obtained, the range $\theta$ is:

$$\begin{aligned}\theta_p &= \pi - \theta_2\\ \theta_k &= \pi - \theta_1\end{aligned} \tag{5}$$

In the case of the *CD* curve, for which a convex thread (blue) is obtained, the range $\theta$ is:

$$\begin{aligned}\theta_p &= \pi - \theta_1\\ \theta_k &= \pi - \theta_2\end{aligned} \tag{6}$$

The parametric description of the arc in plane $y_1z_1$ is shown in Equation (7). In this equation, $y_o$ and $z_o$ need to be substituted with corresponding coordinates of the arc center, while parameter $\theta$ – with the appropriate range (Equation (3), (4), (5) or (6)):

$$\bar{r}_{łuk}^{(1)} = \begin{bmatrix} x_1(\theta) \\ y_1(\theta) \\ z_1(\theta) \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ R\cdot cos(\theta) + y_o \\ R\cdot sin(\theta) + z_o \\ 1 \end{bmatrix} \tag{7}$$

where: *R* – radius of the arc profile; $y_o$, $z_o$ – coordinates of the arc profile center, $\theta$ – parameter ($\theta = \theta_p : d\theta : \theta_k$).

After determining the parametric equation of the tooth profile, a parametric description of the globoid helix [2] should be used. The transition of any point along a globoid helix is described by a homogeneous transformation matrix:

$$M = \begin{bmatrix} \cos(\varphi_1) & \cos(\varphi_2)\cdot\sin(\varphi_1) & -\sin(\varphi_2)\cdot\sin(\varphi_1) & a\cdot\cos(\varphi_1)\cdot\cos(\varphi_2) \\ -\sin(\varphi_1) & \cos(\varphi_2)\cdot\cos(\varphi_1) & -\cos(\varphi_2)\cdot\sin(\varphi_1) & -a\cdot\cos(\varphi_1) \\ 0 & \sin(\varphi_2) & \cos(\varphi_2) & a\cdot\sin(\varphi_2) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

where: $\varphi_1$ – worm rotation angle (and parameter), $\varphi_2$ – wormwheel rotation angle (and auxiliary parameter), *a* – distance between worm and wormwheel axes.

In equation (8), dependence $\varphi_2 = \varphi_1 \cdot i$ should be used, which is determined based on the worm gear ratio value:

$$\mathrm{i} = \frac{z_1}{z_2} = \frac{\varphi_2}{\varphi_1} \tag{9}$$

where: $z_1$ – number of worm teeth, $z_2$ – number of wormwheel teeth.

Parametric equation of the tooth side surface of a globoid worm with a no-straight tooth profile is obtained by moving the tooth arc profile along a globoid helix. The radial vector of no-straight worm thread surface can be determined from the equation:

$$\bar{r}_1^{(1)}(\varphi_1, \theta) = M \cdot \bar{r}_{łuk}^{(1)} \tag{10}$$

By introducing equations (7) and (8) to (10), we obtain:

$$\bar{r}_1^{(1)}(\varphi_1, \theta) = \begin{bmatrix} x_1(\theta)\cdot\cos(\varphi_1) - a\cdot\sin(\varphi_1) + a\cdot\cos(\varphi_2)\cdot\sin(\varphi_1) + \\ +y_1(\theta)\cdot\cos(\varphi_2)\cdot\sin(\varphi_1) - z_1(\theta)\cdot\sin(\varphi_2)\cdot\sin(\varphi_1) \\ -x_1(\theta)\cdot\sin(\varphi_1) - a\cdot\cos(\varphi_1) + a\cdot\cos(\varphi_1)\cdot\cos(\varphi_2) + \\ +y_1(\theta)\cdot\cos(\varphi_2)\cdot\cos(\varphi_1) - z_1(\theta)\cdot\sin(\varphi_2)\cdot\cos(\varphi_1) \\ a\cdot\sin(\varphi_2) + y_1(\theta)\cdot\sin(\varphi_2) + z_1(\theta)\cdot\cos(\varphi_2) \\ 1 \end{bmatrix} \tag{11}$$

The equation (11) defines the range of the worm thread by parameter $\varphi_1$. It changes from the initial value $\varphi_{1p}$ to the final value $\varphi_{1k}$ with the step d$\varphi_1$.
Equation (11) makes possible to obtain a concave or convex surface of the corresponding tooth flank (Fig 3).

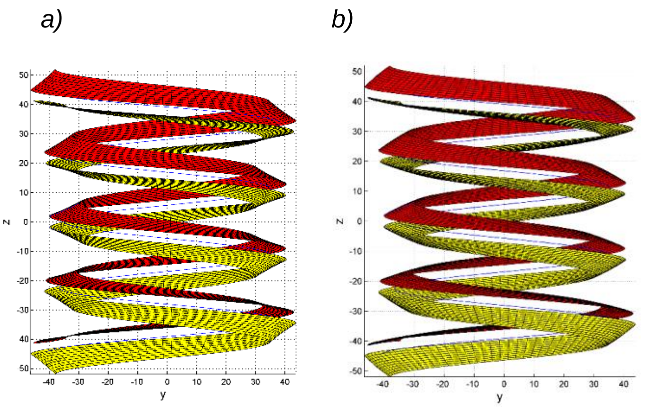


Fig 3. Flanks of globoid worm with *a)* concave and *b)* convex profile

## Conclusions

The parametric equation of the tooth side surface of the globoid worm with convex or concave tooth profile can be used to perform the mathematical model of wormwheel and CAD models of the gear, as well as to perform the gear analysis (e.g. contact pattern analysis or FEM analysis).

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