# Determination of distributions of fatigue crack length by Monte Carlo method

## Wyznaczanie dystrybuant długości pęknięcia zmęczeniowego metodą Monte Carlo

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**Described is the determination of random distributions of the fatigue crack length by the Monte Carlo method and the Bogdanov-Kozin model. Input data needed to determine the distributor were obtained by simulation of fatigue crack growth using the Paris-Erdogan model.**
**KEYWORDS: fatigue, probabilistic models, Bogdanov-Kozin model, Paris-Erdogan model, Monte Carlo method**

On 3 June 1998, in German Eschede, train running at a speed of over 200 km/h hit the railway viaduct, which then collapsed on one of the wagons. 101 people were killed, 105 were injured. This was the most serious high speed train accident ever and was caused directly by the derailment composition on a traveling track due to damage to the wheel rim due to the fatigue [12].

Fatigue is the most common construction materials cause damage to machine components [1]. Critical damage to the machine elements usually appear suddenly and unexpectedly. The primary reason for this is the complicated nature of the processes of fatigue and the fact that in the initial phase of the fatigue, processes occurring in the machine elements are difficult to see.

Fatigue processes are the result of the weave of many complex phenomena characterized by large random scattering which is difficult to describe in a deterministic way. Therefore, to describe the fatigue processes are increasingly being used probabilistic models. Fatigue life is determined on the basis of generally expensive and time-consuming research. They are carried out on finished components, after the design process. Due to these disadvantages, it is advisable to look for methods that can help determine the fatigue life of the element at the design stage. In many cases, physical fatigue tests can be replaced by appropriately constructed a mathematical model. But such models are characterized by high computational complexity, which is why computers are recommended for calculations. Development of numerical methods for determining fatigue life seems to be a perspective direction of scientific research.

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The fatigue process can usually be divided into three stages. In the first stage, local plastic deformations occur and local strengthening and weakening of the element occurs along with them. In the second stage generates micro-cracks, which number increases with the number of load cycles. On the third and last step is the development and linking of micro, thereby macro-cracks arise leading to the complete destruction of the item. Cracks occur mainly on the surface or in the top layer, but all kinds of defects from the manufacturing process can cause cracking on both the surface and subsurface layers. The effect of fatigue processes is complete rupture, also called a breakthrough fatigue. Fatigue breakthrough is the result of the development of surface and subsurface foci perpendicular to the direction of elongation.

The expansion of the fatigue slit can be described by the general equation of fatigue cracking:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = F(a, S, C, \theta, \zeta) \tag{1}$$

where: $a$ – current fatigue crack length, $N$ – number of load cycles (or time) corresponding to the crack length, $S$ – stress in the material resulting from variable loads, $C$ – material properties in general, $\theta$ – temperature, $\zeta$ – other factors impact on the growth gap fatigue.

Typically, formulas describing the increase of fatigue cracks are obtained experimentally. The most famous and most popular is the Paris-Erdogan equation:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C(\Delta K)^m \tag{2}$$

where: $C$, $m$ – material constants, $\Delta K$ – stress intensity factor range.

The fundamental problem in this equation is determination of material parameters $C$, $m$. They depend on many factors such as physical and geometrical properties of the material, the temperature, the frequency and the range of stresses. They must therefore be designated separately for each case under consideration. In contrast, the stress intensity range *ΔK* is usually determined by numerical methods, mainly by the finite element method [2].

## Determination of random distribution of fatigue crack length by Monte Carlo method

Input data were obtained from the simulation of fatigue crack growth simulations using the Paris-Erdogan model. The result of the simulation was a bundle of curves showing the increase in fatigue crack length depending on the number of load cycles. Subsequent curves in the beam were generated by stepwise changing the value of parameter *C* from the Paris-Erdogan equation.

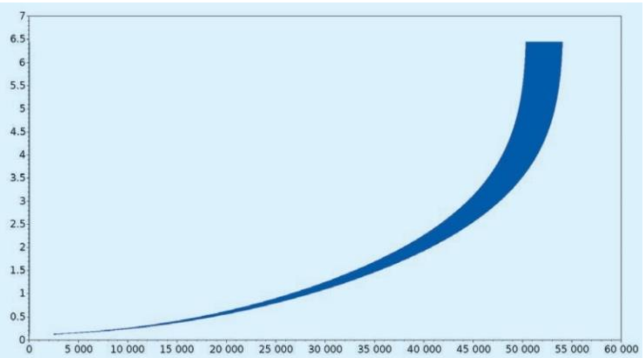

Fig. 1. Pattern of the fatigue crack growth curve obtained using the Paris-Erdogan model

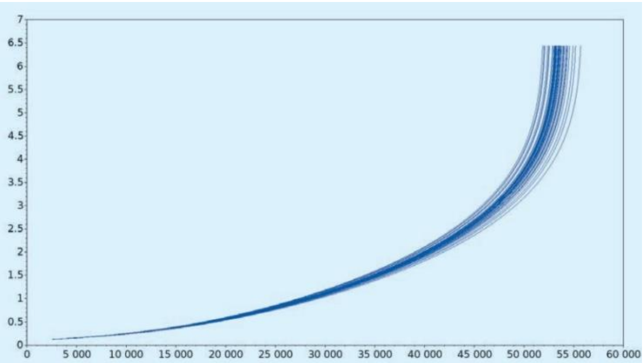

Fig. 2. Pattern of the fatigue crack growth curve obtained from the Paris-Erdogan model by the Monte Carlo method

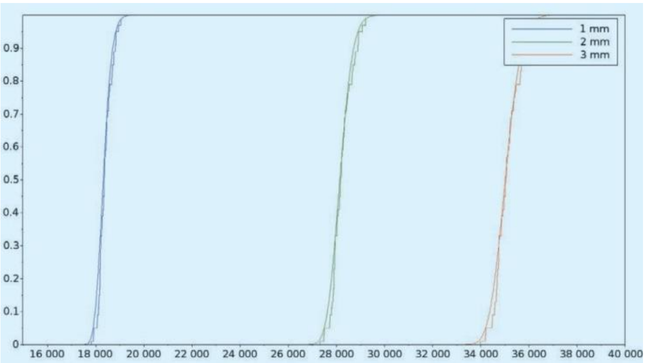


Fig. 3. Pattern of the fatigue crack distribution curve obtained by Monte Carlo method and Bogdanov-Kozin model

The generated beam contained 1000 curves. The graph (fig. 1) shows all generated curves for a single data set. After statistical analysis, it was found that the random variables associated with the cracks lengths obtained from the Paris-Erdogan model are exponential distributions and logarithmic distribution is to be reported according to literature [9]. Therefore, in order to obtain the correct distribution of random variables, a Monte Carlo-based algorithm was chosen, which selects the appropriate curves from the input beam in such a way that the distributions of the random variables assigned to the crack length data have logarithmic-normal distributions. This algorithm also takes into account that these distributions in the case of an input beam are exponential. Fig. 2 shows the result of the developed algorithm. To maintain the readability of the graph, only 50 curves were selected. While the algorithm itself in normal operation of the input curve 1000, selects 200. The selected number of curves was determined by analysis of the histogram obtained by the Monte Carlo simulation.

Based on the input curves, the fatigue gap length distribution was also generated using the Bogdanov-Kozin model. This model is described more fully in [4–8, 10]. The obtained distributions for the three crack lengths: 1 mm, 2 mm and 3 mm are shown in fig. 3. The Bogdanov-Kozin distribution models were presented using smooth curves, and those derived from the Monte Carlo method – using stepped curves.

## Conclusions

Fatigue crack lengths obtained using the Bogdanov-Kozin model and the Monte Carlo method are highly compatible. The main drawback of the Monte Carlo method is high demand for computing power. Bogdanov-Kozin model is characterized by lower demand for computer power and the resulting distribution well describes the distributions of random variables on the individual lengths of cracks. In addition to the Monte Carlo method and the Bogdanov-Kozin model, Weibull distribution was used to generate the distribution. (to keep the graph clear, it was not shown in fig. 3). The algorithm presented was included in the software described in [8, 11]. This software will be further developed.

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