Example of application of two-stage method in calculations of trusses loaded in non-symmetric way

Przykład zastosowania metody dwuetapowej w rozwiązywaniu kratownic obciążonych niesymetrycznie

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DOI: https://doi.org/10.17814/mechanik.2017.7.80

The paper presents analysis of results obtained by application of two-stage method of calculation of the statically indeterminate systems for selected type of plane trusses loaded in nonsymmetrical way. At each stage of this method are calculated values of forces acting in members of the statically determinate trusses, schemes of which are determined by suitable reduction of number of members, which number is equal to statically indeterminacy of the basic truss. In particular stages the outer load forces are of half values of load of the basic truss and they are applied to suitable nodes of the intermediate trusses. Geometric parameters referring to the clear span and construction depth of the considered trusses are in each stage the same like in the basic truss. Final values of forces calculated in the statically indeterminate truss are resultants of forces determined at each stage for appropriate members of statically determinate trusses.

KEYWORDS: truss, statically indeterminate system, method of calculation

The two-stage method of calculation statically indeterminate trusses was created during the initial static analysis of complex tension-strut structures [1]. It uses the simple principle of superposition, which states that the result of a given force can be defined as the sum of the effects of the component forces [2-5]. The inspiration for the emergence of the two-stage method was the images of the deformation of the flat forms of tension-strut structures, which were statically indeterminate systems whose top and bottom chords were made with tension members. If the initial pre-stressing is insufficient and the load forces are too big, the tension members located e.g. in the top chord do not participate in the transmission of forces to the support points. The number of inactive members in this case is equal to the degree of static indeterminacy consistency of the primary truss. This observation has led to the following question: can the statically indeterminate truss be calculated in two stages, using one of the methods applied to calculate the statically determinate trusses, calculating at each stage separately a triangular truss with the appropriate static scheme? The answer to this question is a two-stage method that uses the superposition principle in its procedure [6]. The correctness of accepted assumptions was checked by using this method of calculating the trusses and comparing the results with the results obtained by using appropriate computer software.

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General scheme of the calculation process

In these calculations, flat trusses, consisting of square modules separated by vertical struts, were considered. In this work, a similar truss is statistically indeterminate system, but has a central horizontal chord located at half its height (fig. 1a). It is a truss made of 16 nodes (w = 16) and 33 members (p = 33).

The internally statically determinate truss must meet the following condition:

$$p = 2 \cdot w - 3 \quad (1)$$

This means that for the number of nodes w = 16, the maximum number of members forming the truss is:

$$29 = 2 \cdot 16 - 3 \quad (2)$$

The truss of the diagram shown in fig. 1a is constructed of 33 members and is therefore a four-fold statically indeterminate system. To make it statically determinate truss, four appropriate members must be removed.

It was decided that the four horizontal members of the top chord would be removed in the first stage and the loading forces, equal to half of the load values of the base truss, would be applied to the same top chord nodes as in the primary truss (fig. 1b). At the second stage, four bottom chord members will be removed, and the load-balancing halves will be applied to the bottom chord nodes lying on the respective force lines in the primary truss (fig. 1c). Their positions along these lines are justify by the rules of the calculus of vectors and the feature of forces as linear vectors.

In addition, the proposed procedure results directly from the three basic equilibrium conditions for any coplanar force system:

$$\Sigma P_{ix} = 0 \quad (3)$$

$$\Sigma P_{iy} = 0 \quad (4)$$

$$\Sigma M_i = 0 \quad (5)$$

These three necessary equilibrium conditions sufficiently justify the desirability of applying the superposition principle in the two-stage method of calculation of statically indeterminate trusses.

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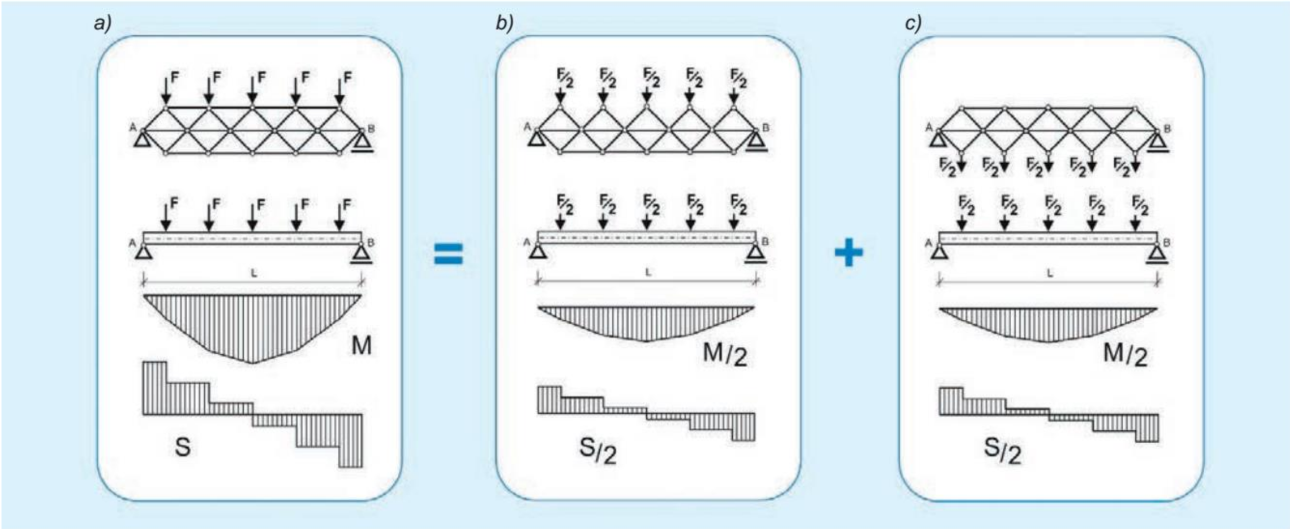

Fig. 1. Diagram of procedure in two-stage method of calculation plane statically indeterminate trusses together with illustrative analogy to bending beams

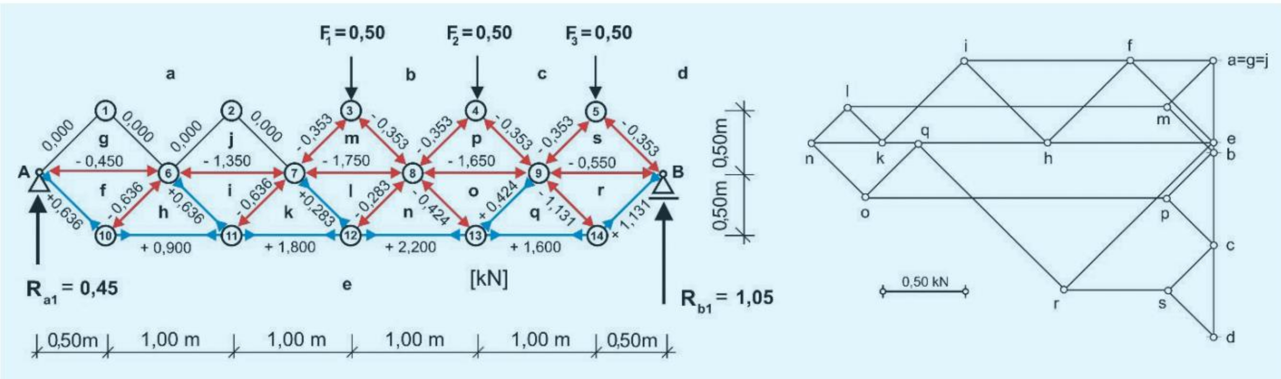

Fig. 2. The values of the forces in members, calculated at the first stage of calculations performed by two-stage method

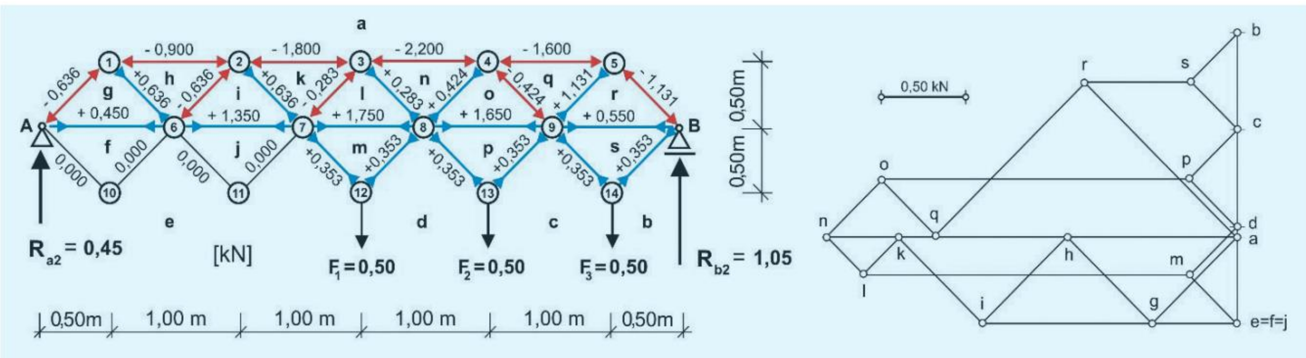

Fig. 3. The values of the forces in members, obtained at the second stage of calculations performed by two-stage method

Calculating the truss loaded in asymmetric way

This section presents the results of the two-stage computation, which were compared to the results obtained for the same truss but loaded in the asymmetric way, with the identical force values. The geometry of the members of the considered truss is identical to that shown in fig. 1a, but it is unbalanced with only three forces applied to three nodes of the top chord located to the right of the truss in the vicinity of the support B (fig. 2). The clear span of the examined truss is 5.00 m and its construction depth is 1.00 m. The truss is loaded with three forces, each of which has a value of 1.00 kN.

Following the two-stage procedure, the first chord was removed and the concentrated forces, each of value equal to 0.50 kN, were applied at the first stage to the previously indicated top chord nodes. Calculations were made using the Cremona's method, and the values of the forces obtained for each of the members were shown (along with the complete Cremona's polygon of forces) on the truss pattern (fig. 2). At the second stage, also four members were removed, but this time from the bottom chord, and the forces of 0.50 kN were applied to the corresponding nodes of the chord. As before, the calculation process was carried out using the Cremona's method, and its results are illustrated in fig. 3.

Using rules of calculus of vectors, the forces in members of the examined base truss are determined successively, which are the calculated forces at each stage for the members located between the nodes with the same ordinal numbers. Since the first stage removes, for example, the member between the nodes of the top chord, numbers 3 and 4 (fig. 2), therefore the value of the force acting there is zero. At the second stage, this strut is present between nodes 3 and 4 (fig. 3), and the calculated compressive force acting therein is -2.20 kN. Therefore, the resulting force value in this member of the considered triangular statically non-determinable is -2.20 kN.

Values of forces acting on all truss members, calculated by two-stage method, are shown in fig. 4a. For an exemplary cross brace between 4 and 8 nodes, the value of the compressive force acting on it, calculated at the first stage, is -0.335 kN (fig. 2), while the force determined in the same rod at the second calculation stage is a tensile force 0.424 kN (fig. 3). The resulting value of the force acting in this member calculated by the two-stage method is therefore approximately 0.07 kN (fig. 4a), which means that there is a small tensile force therein.

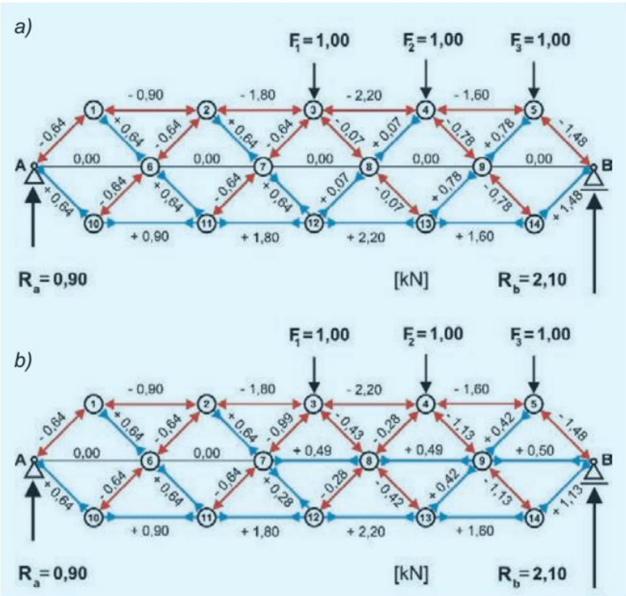

Fig. 4. Values of forces in the members of the tested primary truss calculated: *a*) by two-stage method, *b*) by means of a computer program

It should be noted that the method does not take into account the stiffness of the members joining the truss nodes. From the preliminary analysis of the obtained force values, it follows that the general arrangement of forces, in particular as regards their nature, is in line with the predictions, also concerning the variance of the forces present in the area of the considered truss system.

In order to verify the received values of forces, a truss with the same static pattern (fig. 4b) was calculated in the Autodesk Robot Structural Analysis Professional 2016 computer software, also taking into account the different rigidity of the individual elements in the connecting nodes.

The tested truss also had a clear span of 5.00 m and a construction depth of 1.00 m and was loaded and supported in the same way as the primary truss. It was assumed that the truss was constructed of tubular steel members with Young's modulus E = 210 GPa, a circular cross-sectional diameter of ⌀30.00 mm and a wall thickness of 4.00 mm. The force values obtained by this path and their distribution in the space of the calculated truss are shown in fig. 4b.

On the basis of the comparison of the results obtained by two methods, it can be stated that in the areas of the trusses there is full consistency of the results, and in other regions the values of forces calculated for the same rods differ considerably. The calculated forces acting in the top and bottom chords are identical regardless of the method used. Thus, the value of the force acting e.g. in the member between the nodes 3 and 4 calculated by the computer method (fig. 4b) is -2.20 kN and is identical to the value of the force acting in the same bar, calculated by the two-stage method (fig. 4a). Significant differences also apply to the size of the forces acting in the cross braces. The value of the force defined by the two-stage method in the member between nodes 4 and 8 is 0.07 kN (fig. 4a), which means that it is a tensile force. In turn, the force acting in the same cross brace of truss, calculated by means of a computer program, is -0.28 kN and therefore is a compressive force. Similarly, large differences in the force values were observed in the members forming the central chord running between the support nodes *A* and *B*, that is, as before in the region of occurrence of forces with very small values or equal to zero.

Conclusions

Values of forces obtained in the Autodesk Robot Structural Analysis Professional 2016 computer program are accurate, because the program uses complex computational routines suitable for statically indeterminate systems. The two-stage method is an approximation, but the accuracy of its results may increase significantly in the future by using the appropriate coefficients determined individually for each node and taking into account the different stiffness of the members joining in that node.

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