How to cite this article:
Authors: Magdalena Zielińska, Grzegorz Zboiński
Title of article:"Adaptacyjna analiza struktur o złożonym opisie geometrycznym i mechanicznym z wykorzystaniem przejściowych elementów skończonych" („ Adaptive analysis of structures of complex geometrical and mechanical description with use of the transition finite elements")
Mechanik, Vol. 91, No. 7 (2018): pages 570-572
DOI: https://doi.org/10.17814/mechanik.2018.7.87

Adaptive analysis of structures of complex geometrical and mechanical description with use of the transition finite elements

Adaptacyjna analiza struktur o złożonym opisie geometrycznym i mechanicznym z wykorzystaniem przejściowych elementów skończonych

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This work concerns inclusion of the transition finite elements in adaptive analysis of structures of complex mechanical and geometrical description. The paper addresses the influence of modifications of the transition model on the numerical solution. The ability to remove the internal boundary layer and the effectivity of error estimation and adaptation are analyzed.
KEYWORDS: transition elements, adaptivity, finite element method, convergence

The research concerns the adaptive analysis of complex structures. These are structures with complex mechanical and geometric description. Geometric complexity consists in the presence of geometric parts of various shapes (solid, thick- and thin-walled, and transition ones). Solid parts are spatially expanded fragments. Shell parts are fragments in which two longitudinal dimensions are definitely larger than the third (transverse) dimension - . The transition parts have a shape that allows the combination of solid and shell parts.

Mechanical complexity, understood as the complexity of a mechanical model, refers to a structure for which more than one mechanical theory (model) is used. The mechanical models that underlie the considerations are: the model of the three-dimensional elasticity (3D), the first order shell model (RM), and transition models (TR). The latter models allow combining three-dimensional theory with first-order shell model.

In the general case, the division of the structure into geometric parts does not have to coincide with the domains described by different mechanical models [1, 2]. This means that the first-order shell model and the model of the three-dimensional elasticity can be used in the shell geometry part, and the transition model or three-dimensional model in the transition geometry part. In contrast, the solid part requires the use of the model of three-dimensional theory of elasticity.

The models used in this work are adaptive. Adaptation of the *p*-type is possible in them thanks to the so-called hierarchical approximation and the corresponding shape functions of incremental character, defined independently in vertices, on edges and on the sides of elements. In turn, the *h*-type adaptation requires the use of the so-called constrained approximation. Changes to the model encoded in the element are possible due to introduction of the hierarchical approximation with a mixed two-dimensional/three-dimensional character. Details of all the approximations used are given in [1].

Transition models

Transition models and adaptive finite elements considered in this work may be of a solid-to-shell or shell-to-shell type. In the first case, they serve to connect the three-dimensional model of the theory of elasticity with the first-order shell model. In the second case, they are used to join the hierarchical shell models with the first-order shell model. In both applications, the structure and algorithm of the element are the same.

The classical transition model allows combining elements described with the three-dimensional theory of elasticity and elements corresponding to the first-order Reissner-Mindlin shell theory. This results in continuity of strain between the basic models, however with high gradients at the interface between the transition (TR) and shell (RM) models. The classic model does not allow for continuity of stress at this interface. Details of the algorithm and numerical tests of this model and element are extensively presented in [1, 3].

During the research, a modification of the transition model was proposed [4]. The modified model still guaranteed the continuity of transverse deformations at the boundary of the transition model, unfortunately with high gradients at the point of connection between the TR and RM models. The changes allowed for continuity in the stress field. They consisted in introducing a corrective transition function to the

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computational algorithm. The changes are described in detail in [5].

In the last, most enhanced transition model presented in [6], stress continuity at the model boundaries was maintained. In addition, a continuous change in the kinematic assumptions of the Reissner-Mindlin theory in the transition zone was assumed. The introduced changes are described in detail in [7].

Numerical research

The structure analyzed at work is a symmetrical quarter of a plate supported by four rectangular columns (fig. 1). This structure may represent a fragment of the supporting structure. It consists of a solid (column), shell (plate) and transition part (connection of a plate and a column).

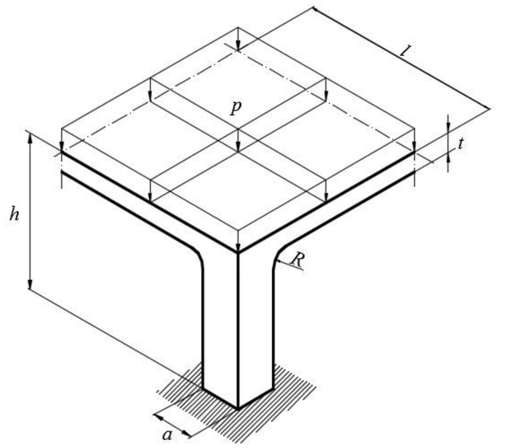

Fig. 1. Symmetrical quarter of plate construction supported by four columns

The following longitudinal dimensions of the structure were assumed: l = 4.1415 m, thickness t = 0.1 m. The height of the column with the plate is h = 1.4 m, and the width of the square section of the column is a = 0.3 m. The transition part between the column and the plate is limited by a quarter of a circle with a radius of R = 0.2 m. A normal, distributed load is applied to the thin-walled part of the structure with a value of $p = 4 \cdot 10^6$ N/m^2. The bottom end of the column is fixed. It was assumed that all parts of the analyzed structure are made of elastic material with Young's modulus $E = 2.1 \cdot 10^{11}$ N/m^2 and Poisson's ratio v = 0.3.

Use of transition elements in the adaptive analysis of complex structures

An assessment of the ability of the transition elements to remove the internal boundary layer will be carried out.

a) b)

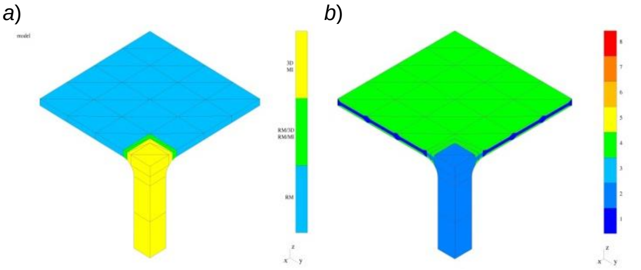

Fig. 2. Symmetrical quarter of the supporting structure – initial mesh of the adaptation process: a) models, b) degrees of approximation

The models used and the division of the structure into elements are presented in: fig. 2 – the mesh and initial models in both analyzed variants of transition elements, figs. 3-4 – the meshes and final models.

■ **Ability to remove the inner boundary layer**. Analyzing the results of numerical research, one can observe a greater ability of the improved model to remove the internal boundary layer than the classical model. In the same way, it applies to the initial and final meshes. This is confirmed in fig. 5 and fig. 6.

a) *b)*

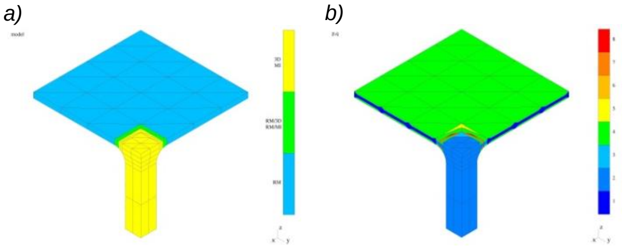

Fig. 3. Final mesh of the adaptation process – a classical transition model used: a) models, b) approximation orders

a) *b)*

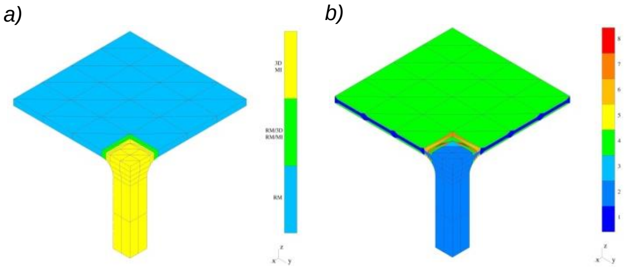

Fig. 4. Final grid of the adaptation process – an enhanced transition model used: a) models, b) approximation orders

The first of the drawings (fig. 5a) reveals the occurrence of the internal boundary layer at the interface of the shell model and the classical transition model in the initial mesh. A visible boundary between these models also exists in the case of the final mesh (fig. 5b). In turn, the application of the enhanced transition model on the initial mesh (fig. 6a) causes the disappearance of the clear boundary between the mentioned models. This also takes place in the final mesh (fig. 6b).

Referring to all drawings, it should be noted that they also reveal jumps in stress distributions caused by discretization, e.g. at the boundary of the column and thin-walled part (fig. 5, in the vicinity of the point with the minimum stress value on the initial mesh, marked with the symbol x).

a) *b)*

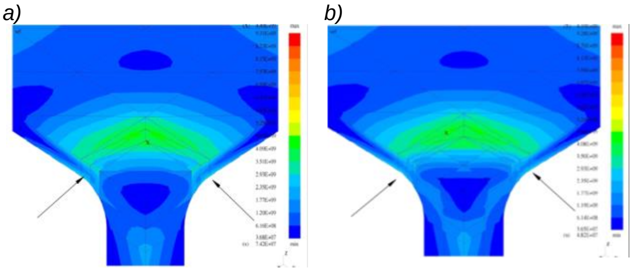

Fig. 5. Effective stresses – a classical transition model: a) initial mesh, b) final mesh

a) *b)*

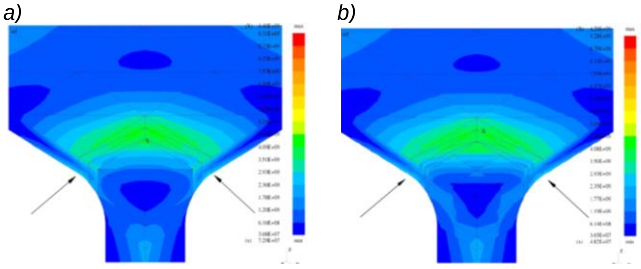

Fig. 6. Effective stresses – an enhanced transition model: a) initial mesh, b) final mesh

Convergence of the adaptation process. The effectiveness of the adaptation process was assessed on the basis of the results obtained in the three-step process of *hp* adaptation. It was performed assuming the admissible value of the target error on the final meshes $\gamma_T = 0.05$. The value of the ratio of the estimated error from the intermediate mesh and the estimated error from the final mesh was assumed as $\gamma_I/\gamma_T = 2.0$.

Fig. 7 shows the relative, estimated values of the local and global, approximation errors in the case of using classical elements in the transition zone. The level of the estimated global error on the initial mesh is *avr* = 0.325, while on the final mesh *avr* = 0.168.

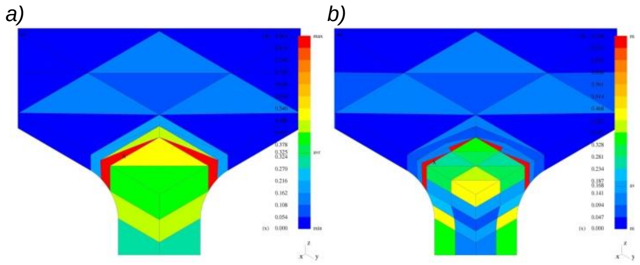

Fig. 7. Approximation error – classical transition model: a) initial mesh, b) final mesh

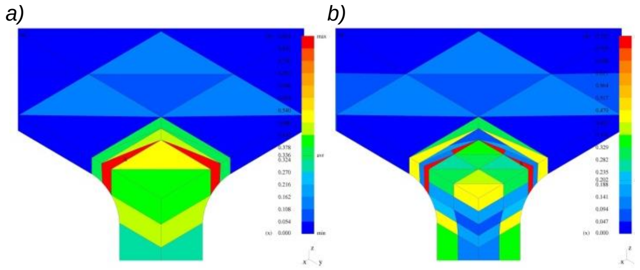

Fig. 8. Approximation error – enhanced transition model: a) initial mesh, b) final mesh

Fig. 8 shows that the analogous values in the case of the model taking into account the enhanced transition element were: *avr* = 0.336 on the initial mesh and *avr* = 0.202 on the final mesh. The obtained results concern a mesh different from that obtained in the case of using a classical transition element.

The presented adaptation process, controlled by the estimated error values from figs. 7-8, corresponds to the convergence curves representing the level of true error as a function of the increasing number of degrees of freedom *N* (fig. 9). The convergence curve corresponding to the classical transition element is dark blue. The curve corresponding to the application of the enhanced element is light blue.

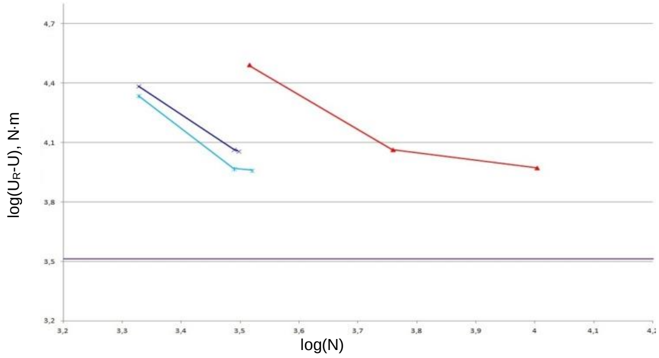

Fig. 9. Convergence curves for the same initial mesh – different levels of target error

A more effective reduction of the true error level results from a higher level of error estimates when using an enhanced element.

The curves obtained for the complex models of structures can be compared with a red curve corresponding to a totally three-dimensional model. The three-dimensional model requires much more (than both complex models) degrees of freedom to get the same or similar value of the global error.

This example shows the superiority of complex models, including shell elements and transition elements, over analysis using three-dimensional elements. The reason for this is the phenomenon of the boundary layer occurring in thin-walled parts, modeled using three-dimensional model.

Conclusions

Considering the effectiveness of the adaptation process in the example of the plate structure supported by four columns, it should be noted that obtaining the same level of errors from the basic three-dimensional model and the complex model including the improved transition element requires, in the latter case, fewer degrees of freedom. In the example concerning the supported plate structure, the number of degrees of freedom of the complex models was much smaller than for the three-dimensional model.

This perfectly justifies the use of complex models instead of the basic model of the three-dimensional theory of elasticity, if only the error level of approximation is considered as the criterion.

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Translation of scientific articles, their computer composition and publishing them on the website www.mechanik.media.pl by original articles in Polish is a task financed from the funds of the Ministry of Science and Higher Education designated for dissemination of science.

Ministry of Science and Higher Education
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