Analysis of a planetary gear modeled with a contour graph taking into account the method of parametric play structures

Analiza przekładni planetarnej zamodelowanej grafem konturowym z uwzględnieniem metody struktur rozgrywających parametrycznie

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Previous applications of the graph theory concerned the modeling of gears for dynamic analysis, kinematic analysis, synthesis, structural analysis, gearshift optimization and automatic design based on so-called graph grammars. Some tasks can be performed only by using methods resulting from a graph theory, e.g. enumeration of structural solutions. The contour plot method consists in distinguishing a series of consecutive rigid units of the mechanism, forming a closed loop (so-called contour). At a later stage, it is possible to analyze the obtained contour graph as a directed graph of dependence. The work presents an example of the use of game-tree structures for describing the contour graph of a planetary gear. As a result of the decomposition of the graph in the dependence on each of the vertices, game-tree structures are obtained, which allow calculate algorithmically.
KEYWORDS: contour graph, planetary gear, graph theory, optimization, game-tree structures

The methods of graphs and structural numbers have long been known in mechanics. There have been many works on the use of graph theory in the study of dynamics of systems, both in the field of analysis and synthesis of complex mechanical systems [1-5]. Unlike graphs, dendrite-tree structures do not have cycles, but may have a different number of initial vertices. Therefore, a different approach has been developed as a translation of the directed dependency graph, among others, into parametric play structures [6]. This approach differs from literature studies related to control systems, analysis of the importance of structural and / or operational parameters, and analysis of transmissions previously modeled using other types of graphs (Freudenstein, contour, Hsu, etc.) [7-10]. The purpose of the parametric trees is to systematize the decision process and the space of possible states of the analyzed system, e.g. in optimizing the parameters of the transmission operation.

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Graphic-theory models of planetary gear

The objectives of the modeling of gears were diverse, including: dynamic analysis, kinematic analysis, synthesis, structural analysis, enumeration, gearshift optimization and automatic design based on so-called graphical grammar. The advantage of this modeling is that graphical models are solved in an algorithmic way. The methods of analyzing planetary gears include methods such as Hsu [8], Freudenstein [9] and Marghit [10].

Model of a planetary gearbox built on contour graph

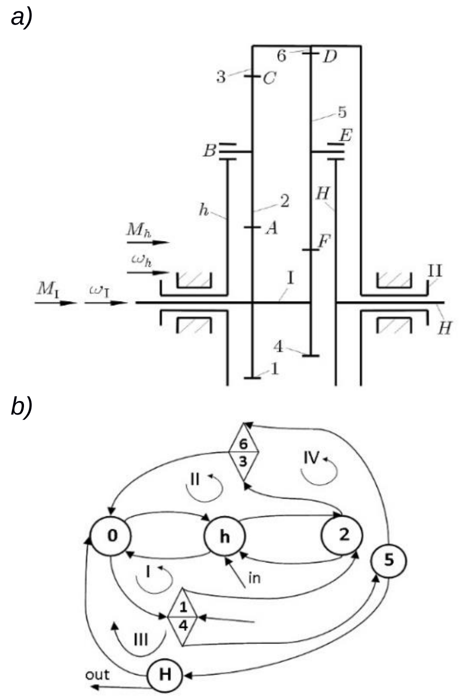

Fig. 1. Example planetary gearbox: a) functional diagram, b) contour graph of gear. It was adopted: *1, 2, ..., 6* - solar wheels, planetary gear wheels and inner gear rings; *h, H* - yoke; *A, B, ... F* - characteristic points; *ω, M* - velocities [4]

Fig. 1 shows a functional diagram and a contour graph of an exemplary planetary gear. It was adopted: *1, 2, ..., 6* - solar wheels, planetary wheels and rings with internal gearing.

Graphical analysis of this gear is possible [3, 4]. For the considered gear, the following is assumed: z_1 = 15, z_2 = 24, z_3 = 63 (-63), z_4 = 18, z_5 = 21 and z_6 = 60 (-60) (where: z_i - number of teeth on the *i*-th circle, *i* = 1, 2, ..., 6), *m* = 2 mm (*m* - wheel module), where the negative numbers of teeth are appropriate for the method. To create a full graph, one must analyze all contours. For the graph of fig. 1, the contour codes can be written:

I: 0→1→2→h→0; ***II***: 0→*h*→2→3→0;
III: 0→*4*→5→H→0, ***IV***: 0→*4*→5→6→0

Each contour generates equations referring to rotational speeds [3, 4]:

$$\begin{cases} -r_A\omega_1 - r_A\omega_{21} - r_A\omega_{h2} - r_A\omega_{0h} = 0 \\ r_A\omega_{21} + r_B\omega_{h2} = 0 \\ -r_C\omega_{h0} - r_C\omega_{2h} - r_C\omega_{32} - r_C\omega_{03} = 0 \\ r_B\omega_{2h} + r_C\omega_{32} = 0 \\ -r_E\omega_{40} - r_E\omega_{54} - r_E\omega_{H5} - r_E\omega_{0H} = 0 \\ r_F\omega_{54} + r_E\omega_{H5} = 0 \\ -r_D\omega_{40} - r_D\omega_{54} - r_D\omega_{H5} - r_E\omega_{0H} = 0 \\ r_F\omega_{54} + r_E\omega_{H5} = 0 \end{cases} \tag{1}$$

The system was solved by eliminating undesirable relative speeds. Assuming that the circles are cylindrical, the following relationships are obtained:

$$r_i = \frac{d_i}{2} = \frac{z_i \cdot m}{2} \tag{2}$$

where: d_i - diameter of the *i*-th wheel, m_i – module of the *i*-th wheel, z_i - number of teeth on the *i*-th wheel, *i* = 1, 2, ..., 6.

In the work [3, 4], the gearbox of fig. 1 is analyzed by the classic Willis method. Additionally, the torque acting on the gear unit can be determined using the contour plot method.

Game-tree structures

A specific type of coherent graphs (without cycles) are trees. Many examples of trees provide logical structures, such as multi-valued logic trees, dendrites, multi-player games [11-15]. The directed dependency graph [13, 14] defines the analytical expressions for this graph, and thus its analytical model. Structured tree structures play out as a result of the distribution of directed graph dependence on each of the vertices. There are also extensively developed graph applications in hydraulic property research. multiple vertex numbering and complexity coefficient of parametric game structures.

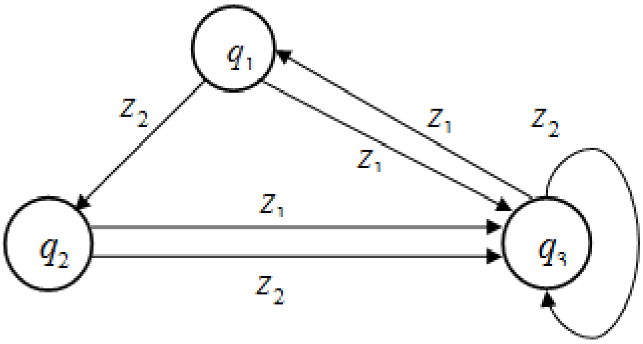

Fig. 2. Directed graph of parametric game

Fig. 2 shows an example of a directed graph of parametric game.

As a result of the decomposition of the graph from the selected initial vertex q_1 in the first stage, a tree structure with G_i^+ cycles is obtained:

$$G_1^+ = ({}^0 q_1({}^1 z_1 q_3({}^2 z_1 q_1, z_2 q_3)^2, z_2 q_2({}^2 z_1 q_3, z_2 q_3)^2)^1)^0 \tag{3}$$

then the general tree structure of parametric game G_1^{++}:

$$G_1^{++} = ({}^0 q_1({}^1 z_1 q_3({}^2 z_1 q_1^1, z_2 q_3^1)^2, z_2 q_2({}^2 z_1 q_3({}^3 z_1 q_1^2, z_2 q_3^1)^3, z_2 q_3({}^3 z_1 q_1^2, z_2 q_3^1)^3)^2)^1)^0 \tag{4}$$

The algorithm of distribution of the dependence graph on the parametric game structure together with the obtained structures has been presented, among others, in [15].

Application of parametric game structures in planetary gear analysis modeled with contour graph

For the contour graph of fig. 1, corresponding to the analyzed gear, a set $D(G_i^{++})$ can be determined for the parametric game units corresponding to six planetary circles: *1, 2, 3, 4, 5, 6*:

$$D(G_i^{++}) = \{G_0^{++}, G_2^{++}, G_{\{1,4\}}^{++}, G_{\{3/6\}}^{++}, G_5^{++}\} \tag{5}$$

Parametric tree structures of parametric game are created after the removal of cycles from primary trees created during the initial phase of the graph. The analytical expressions (6-7) describe exemplary tree structures that play parametrically G^+_2, $G^+_{\{6/3\}}$:

$$\begin{aligned} G_{P2}^{++} = & ({}^0 2({}^1[I]\cdot h({}^2[II]\cdot 2^1, [I]\cdot 0({}^3[II]\cdot h^1, \\ & [I \wedge III]\cdot[1/4({}^4[I \wedge IV]\cdot 2^2, [III \wedge IV]\cdot 5 \\ & ({}^5[III]\cdot H({}^6[III]\cdot 0^1, [IV]\cdot[6/3]({}^7[II]\cdot 0^2)^7)^6)^5)^4)^3)^2, \\ & [II \wedge IV]\cdot[6/3]({}^2[II]\cdot 0)^2)^1)^0 \end{aligned} \tag{6}$$

$$\begin{aligned} G_{\{6/3\}}^{++} = & ({}^0 6/3({}^1[II]\cdot 0({}^2[II]\cdot h({}^3[II]\cdot 2({}^4[I]\cdot h^1, \\ & [II \wedge IV]\cdot\{6/3\}^1)^4, [I]\cdot 0^1)^3, \\ & [I]\cdot 1/4({}^3[I \wedge IV]\cdot 2({}^4[I]\cdot h({}^5[II]\cdot 2({}^6[I]\cdot h^2, \\ & [II \wedge IV]\cdot\{6/3\}^2)^6)^5)^4, [III \wedge IV] \\ & \cdot 5({}^4[III]\cdot H({}^5[III]\cdot 0^2)^5, [IV]\cdot\{6/3\}^3)^4)^3)^2)^1)^0 \end{aligned} \tag{7}$$

Fig. 3 shows parametric game structures from G^{+}_{2}, $G^{+}_{\{6/3\}}$ vertexes for expressions (6) and (7).

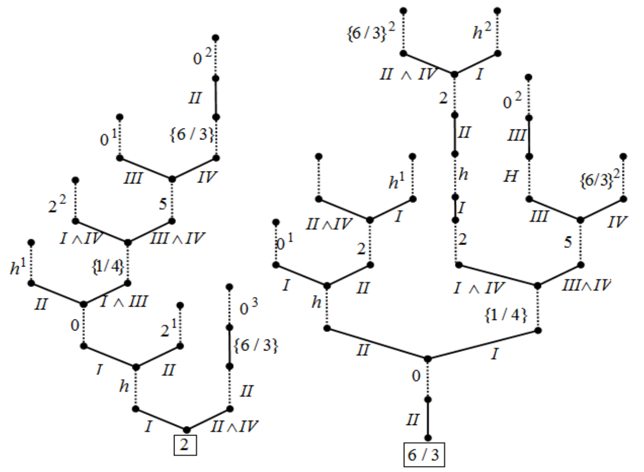

Fig. 3. Parametric game structures from vertices 2 and {6/3}

Parametric game structures from all vertices of the directed graph of dependencies in fig. 1, as well as their detailed analytical description are presented in [16].

The advantages of graphical methods are algorithmic approach to problems and the ability to perform other tasks, e.g. algorithmic finding of excess circles or enumeration of structural solutions. In this perspective, parametric game structures can better represent the algorithmic capabilities of a given gear.

Conclusions

Using the parametric structure method, a better representation of the planetary gear can be obtained in the areas of structural analysis and the analysis of the subordinate expressions:

Structural analysis. Each parametric game structure can describe an independent grouped relational system of preference. This is because the search for solutions on the structure has the properties of the oriented graph [17], and additional multiple vertex numbering allows the indexing (selection) of both vertices and edges to be based on analysis of the incident edge indices with those vertices. In addition, game structures parametrically distinguish decisions regarding individual contours. This is extremely important because, as a result of graphing operations, the identity equations can be found in the parametric structures, and the variable can be determined with more than one dependency.

Analysis of subordinate expressions. An expression describing the degree of subtraction of a given component graph, i.e., the game structure of a parametric representation, is denoted by a pair of parentheses $(^{k} \ldots)^{k}$, inside which the expression being an analytical model is written. A string formed by symbols describing vertices and edges (structural parameters and/or functions) may be subjected to factoring processes, i.e., finding the product form on the basis of the product of sums or decompositions, i.e., the distribution by a set of functions. Analytical writing can be subjected to extraction processes, that is, to express many functions using a set of new functions or to replace one structure with another.

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